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Interaction of an atom with a small dispersive and absorptive dielectric body

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The paper analyzes the interaction of an atomic system with a quantum damped harmonic oscillator. Such an oscillator is the building block in the recently proposed models of bulk dielectrics and may also serve as a simple model of a small dielectric body. Dispersion and losses are taken into account by assuming the oscillator to be coupled to a zero-temperature reservoir consisting of an infinite system of other harmonic oscillators. The Green's function of the atomic system is calculated perturbatively when coupled to the bath. The self-energy of the atomic electron is obtained by the partial resummation of perturbation diagrams, and thus energy-level shifts of both ground and excited states of the atom due to the presence of the oscillator are determined. Corrections to the decay rates are also obtained and analyzed as functions of the distance of the atom from the oscillator, and of the coupling of the oscillator to the reservoir.

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I. INTRODUCTION

The problem of the interaction of atoms with electromagnetic fields in the presence of dielectric bodies and interfaces has attracted a lot of attention over the past three decades (cf., e.g., Refs. [1–11]). Interactions of this kind are particularly interesting in the case of lossy and dispersive dielectrics, since these describe realistic materials. One of the most successful models of such media involves their representation in terms of a system of harmonic oscillators coupled to a reservoir consisting of yet another collection of harmonic oscillators [5]. However, the application of this model even to very simple inhomogeneous systems is nontrivial, and because of a multitude of technical difficulties the approach has mostly been applied only to one-dimensional systems. However, going beyond one spatial dimension is the very complex though powerful formalism employed in Ref. [10]; it facilitated the extraction of expressions for the spontaneous decay rate of an atom near an interface in three dimensions, but energy-level shifts were not discussed. We want to determine both level shift and decay rates, and we shall use a Green's-function approach that is similar in the spirit to that of Ref. [10].

In view of the above-mentioned technical arduousness, it seems appropriate to tackle the general problem by first studying in some detail the simplified problem of an atom interacting with just one harmonic oscillator, and this is the objective of the present paper. There is a twofold motivation for studying this simple model. On one hand, the experience that one gains from it and the methodology developed for it provide useful intuitions for more complex structures. These are, e.g., photonic crystals built of many discrete small dielectric particles, or macroscopic bodies which can be thought of as a continuum of oscillators. To facilitate such generalization we shall model the oscillator by a continuous oscillator field density and shall select the geometry of a single oscillator only later by choosing a pointlike coupling with the electromagnetic field. On the other hand, even such a simple one-oscillator system is a realistic model for a small polarizable body with dispersion and losses (''small'' meaning here much smaller than any characteristic wavelengths involved). Another objective of this paper is to study the effect of the coupling to a reservoir by comparing the results for undamped and damped oscillators (i.e., uncoupled from and coupled to a reservoir, respectively). Again, useful information can be gained from such a comparison, and this can be employed in the analysis of the atom-field interactions in the presence of large dielectrics for finding out what difference losses make. For the case of nonretarded interactions, such a comparison has been performed in Ref. [1].

To forestall confusion we emphasize that we are interested strictly in the van der Waals problem of an atom interacting with external dielectrics. If one were to consider atoms embedded in dielectric materials, then one would have to take local-field corrections into account (see, e.g., Ref. [6]). The formalism developed here is not suited to that problem.

The atom and the oscillator are coupled via their polarizations in the electromagnetic field. In this work we put both the atomic and the oscillator polarizations into the interaction Hamiltonian and treat both of them perturbatively. One could, of course, include the oscillator-polarization term in the zeroth-order Hamiltonian and determine the Green's function of the electromagnetic field that is already corrected for the presence of the oscillator, i.e., to all orders. This is possible because the problem of the interaction of a charged harmonic oscillator with an electromagnetic field is exactly soluble, either by diagonalization of the Hamiltonian [13] or by solving the initial-value problem for the Heisenberg operators [14]. Then, the atom could be considered to interact not with the electromagnetic field and through it with the oscillator, but just with the ''dressed'' electromagnetic field. This approach is in fact the only option for atom-field interactions in the presence of large and optically dense dielectric bodies, where a perturbative treatment of the oscillator-field coupling would not converge. However, apart from technical

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simplicity, the perturbation expansion with respect to both polarizations has merit for a small dielectric body because it treats the two interacting objects (the atom and the oscillator) on the same footing and, thus, leads to elegant and symmetric expressions. Furthermore, this approach allows us to relate to the standard perturbative theory of atom-atom interactions.

The main body of the paper is organized as follows. In Sec. II, we provide the Hamiltonian and free propagators necessary for our Green's-function approach. Section III contains the perturbative calculation of the atomic propagator. We formulate simple Feynman rules for constructing the diagrams of the perturbation expansion. Both the (rather simple) Green's functions and the Feynman rules are discussed at some length since they are provided here in quite a general form, suitable for dielectric bodies of arbitrary type and size. In Sec. IV the energy-level shifts and decay rates of the atom are extracted and analyzed in various asymptotic regimes. Finally, Sec. V contains a summary and a few concluding remarks. The Appendix derives the retarded Green's function for the oscillator when coupled to the bath and examines the analytic properties of its Fourier transform.

The oscillator representing the small body will in the following be referred to as the ''system'' oscillator, while the oscillators that constitute the reservoir will be called the ''reservoir'' or ''bath'' oscillators.

II. FORMULATION OF THE PROBLEM—THE HAMILTONIAN AND THE FREE GREEN'S FUNCTIONS

Our objective in this paper is to describe the interaction of an atom with a damped harmonic oscillator, mediated by the electromagnetic field. We look for the energy-level shifts and modified decay rates of the atom due to this interaction. Thus, the system under considerations consists of the following subsystems.

(i) The atom, which we choose to be located at the point $\mathbf{r}=\mathbf{R}$ and described by the following free second-quantized Hamiltonian

$$H_A=\sum_l E_l c_l^\dagger c_l . \tag{1}$$

Here, c_l and $c_l^\dagger$ are the annihilation and creation operators, respectively, of the atomic electron in level l, and the sum runs over all energy levels of the atom. As we consider a one-electron atom, we have $\Sigma_l c_l^\dagger c_l=1$.

(ii) The free electromagnetic field has the Hamiltonian

$$H_F=\int d^3\mathbf{r}\left(\epsilon_0 E^2(\mathbf{r})+\frac{1}{\mu_0}B^2(\mathbf{r})\right). \tag{2}$$

Since H_F describes the free radiation field, $\mathbf{E}(\mathbf{r})$ is the transverse electric field which is related to the dielectric displacement through $\mathbf{E}=\mathbf{D}/\epsilon_0$.

(iii) The system oscillator and the bath oscillators are described by the Hamiltonian

$$H_{O-R}=\int d^3\mathbf{r}\left[\frac{P^2}{2\mathcal{M}}+\frac{1}{2}\mathcal{M}\omega_0^2 X^2 + \int_0^\infty d\nu\left(\frac{Z_\nu^2}{2\rho_\nu}+\frac{\rho_\nu}{2}\nu^2(\mathbf{Y}_\nu-\mathbf{X})^2\right)\right], \tag{3}$$

where $\mathbf{X}$ and $\mathbf{P}$ are the dipole moment and the conjugate momentum of the system oscillator, and $\mathbf{Y}$ and $\mathbf{Z}$ are the dipole moments and conjugate momenta of the bath oscillators. Strictly speaking, all these variables are densities, and the Hamiltonian actually describes continuous fields of the system and the bath oscillators. We shall pick the geometry of our model dielectric by allowing the atom to interact only with oscillators inside a certain volume. This works fine because oscillators at different points in space are, of course, mutually independent. Our model is similar to the independent-oscillator model of the reservoir developed in Ref. [20], except that we use a continuous field rather than a discrete assembly. The model describes an absorbing dielectric with an absorption band around frequency ω_0. The strength of the dielectric response is determined by the ''mass'' $\mathcal{M}$, as the combination $1/(\mathcal{M}\omega_0^2)$ has the physical meaning of a static polarizability [12]. The bath oscillator masses ρ_ν govern the damping due to the absorption and thus prescribe the precise profile and structure of the absorption spectrum. We shall discuss all this in more detail in Sec. IV B.

The total zeroth-order Hamiltonian is

$$H_0=H_A+H_F+H_{O-R}.$$

It describes the unperturbed system of the atom and the system oscillator when not coupled to each other.

The interaction is mediated by the polarizations of the atom and of the system oscillator in the electromagnetic field. In the electric dipole approximation, the interaction Hamiltonian reads

$$V=-\mathbf{d}\cdot\mathbf{E}(\mathbf{R})-\int d^3\mathbf{r}\, g(\mathbf{r})\mathbf{X}(\mathbf{r})\cdot\mathbf{E}(\mathbf{r}) \quad \text{with}$$

$$\mathbf{d}=-e\sum_{ij}\mathbf{q}_{ij}c_i^\dagger c_j . \tag{4}$$

The electron charge is $-e$ and $\mathbf{q}_{ij}\equiv\langle i|\mathbf{q}|j\rangle$ are the transition amplitudes between the atomic states i and j. The dimensionless coupling $g(\mathbf{r})$ specifies the geometry of the dielectric; it is constant inside and zero outside the dielectric body. For the moment we remain general in our derivation and leave it stand as an arbitrary function of $\mathbf{r}$, but eventually we shall choose it to describe a pointlike dielectric and thus be nonzero only at a single point.

The square of the polarization has been omitted from the interaction Hamiltonian, since it contributes only when the positions of the atom and the oscillator coincide. As regards the coupling between the system and the bath oscillators, we note that this has been included in H_{O-R}, Eq. (3).

In this work we are going to use a Green's-function approach. First, we shall determine the free Green's functions

of the unperturbed system, and then we shall calculate the corrections to them by means of a perturbation expansion. The location of the poles of the corrected atomic Green's function gives the energy shifts and lifetimes of the atom due to the interactions with the dielectric body.

We start by compiling all the free propagators needed for the perturbative expansion. In general, field propagators or ''causal Green functions'' are defined by [15,16]

$$G(\mathbf{x},\mathbf{x}',t,t')=-\frac{i}{\hbar}\langle\Omega|\mathsf{T}(\Phi(\mathbf{x},t)\Phi^{\dagger}(\mathbf{x}',t'))|\Omega\rangle, \quad (5)$$

where $\Phi(\mathbf{x},t)$ is an arbitrary field operator in the Heisenberg picture, $|\Omega\rangle$ is the exact ground state of the system, and T is the time-ordering operator. We note, however, that conventions differ slightly in regard to the presence of $\hbar$. The exact ground state of the system is unknown and, in most cases, accessible only perturbatively. Such perturbative calculations are performed in the interaction picture and use Green's functions with respect to the ground state of the noninteracting system. In our case, the ground state of the noninteracting system is simply the tensor product of the vacuum of the atomic-electron field, the vacuum of the electromagnetic field, the ground state of the system oscillator, and the ground state of the reservoir.

A. Atomic-electron propagator

The atomic creation and annihilation operators $c_l^{\dagger}, c_l$ satisfy the anticommutation relations

$$[c_l,c_m^{\dagger}]_+=\delta_{lm}, \quad [c_l,c_m]_+=0, \quad [c_l^{\dagger},c_m^{\dagger}]_+=0. \quad (6)$$

Since the free atomic-electron Hamiltonian is diagonal, the time evolution of the operators $c_l, c_l^{\dagger}$ is simply

$$c_l(t)=c_l(0)e^{-iE_lt/\hbar}, \quad c_l^{\dagger}(t)=c_l^{\dagger}(0)e^{iE_lt/\hbar}. \quad (7)$$

In order to determine the unperturbed atomic-electron propagator

$$G^{(0)}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle\Omega|\mathsf{T}(\psi(\mathbf{r},t)\psi^{\dagger}(\mathbf{r}',t'))|\Omega\rangle$$

we expand the field operators ψ and $\psi^{\dagger}$ in terms of the creation and annihilation operators,

$$\psi(\mathbf{r},t)=\sum_l c_l(t)\phi_l(\mathbf{r}).$$

The c-number functions ϕ_l are the eigenfunctions of the first-quantized atomic-electron Hamiltonian, i.e., the solutions of the Schrödinger equation. We obtain

$$G^{(0)}(\mathbf{r},\mathbf{r}',t,t')=\sum_{l,m}\phi_l(\mathbf{r})\phi_m^{*}(\mathbf{r}')G_{lm}^{(0)}(t,t')$$

with

$$G_{lm}^{(0)}(t,t')=\frac{-i}{\hbar}\langle\Omega|T(c_l(t)c_m^{\dagger}(t'))|\Omega\rangle.$$

From Eqs. (6) and (7) one quickly finds

$$G_{lm}^{(0)}(t,t')=-i\theta(t-t')e^{-iE_l(t-t')/\hbar}\delta_{lm}.$$

Since this Green's function depends only on the time difference $\tau=t-t'$, we can work with its Fourier transform

$$G_{lm}^{(0)}(E)=\int_{-\infty}^{\infty}d\tau G_{lm}^{(0)}(\tau)e^{iE\tau/\hbar}=\frac{1}{E-E_l+i\varepsilon}\delta_{lm}. \quad (8)$$

In order to make the Fourier integral convergent, we have added a small imaginary shift $-i\varepsilon$ to the energy E_l and thus shifted the pole of $G_{lm}^{(0)}(E)$ away from the real axis into the complex plane. This prescription also ensures the correct causal boundary conditions of $G^{(0)}(t,t')$ for $(t-t')\to\pm\infty$.

B. Electromagnetic-field propagator

Although the Green's functions of the electromagnetic field are well known, they are normally given for the vector potential $\mathbf{A}(\mathbf{r},t)$ [18]. Since we are working with the $\mathbf{d}\cdot\mathbf{E}$ coupling, we require the causal propagator of the $\mathbf{E}$ field instead:

$$D_{ij}^{E}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle 0|\mathsf{T}(E_i(\mathbf{r},t)E_j(\mathbf{r}',t'))|0\rangle. \quad (9)$$

It is straightforward to calculate this propagator either from the causal $\mathbf{A}$ field propagator in a fixed gauge, or, without resorting to gauge-dependent derivations, from the field equation for $\mathbf{E}$:

$$\nabla\times\nabla\times\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\mathbf{0}.$$

When applying the same differential operator on propagator (9), one must take care in applying the time derivative to the time-ordering symbol (see, e.g., Ref. [17]). Noting that

$$\frac{\partial^2}{\partial t^2}D_{ij}^{E}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\left\langle 0\left|\mathsf{T}\left(\frac{\partial^2E_i(\mathbf{r},t)}{\partial t^2}E_j(\mathbf{r}',t')\right)\right|0\right\rangle$$
$$-\frac{i}{\hbar}\delta(t-t')\times\left\langle 0\left|\left[\frac{\partial E_i(\mathbf{r},t)}{\partial t},E_j(\mathbf{r}',t')\right]\right|0\right\rangle.$$

and using the gauge-independent equal-time commutator (cf., e.g., Ref. [19])

$$[E_i(\mathbf{r}),B_j(\mathbf{r}')]=\frac{\hbar}{(2\pi)^3\epsilon_0}\epsilon_{ijn}\int d^3\mathbf{k}k_ne^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')},$$

we find that the causal $\mathbf{E}$ field propagator satisfies the equation

$$(\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times D^E)_{ij}(\mathbf{r},\mathbf{r}',t,t')+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}D^E_{ij}(\mathbf{r},\mathbf{r}',t,t')$$

$$=-\frac{1}{(2\pi)^3\epsilon_0}\delta(t-t')\int d^3\mathbf{k}(\delta_{ij}k^2-k_ik_j)e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}. \tag{10}$$

The right-hand side of this equation contains the transverse δ function because the electromagnetic field is a transverse field and, thus, in order to obtain the propagator, one must invert the operator of its field equation in the subspace of transverse fields.

Equation (10) can be solved easily by Fourier transformation. One finds that in free space the matrix elements of the electromagnetic Green's function D^E_{mn} can be written as

$$D^E_{mn}(\mathbf{r},\mathbf{r}',\omega)=(-\nabla^2\delta_{mn}+\nabla_m\nabla_n)d(|\mathbf{r}-\mathbf{r}'|,\omega), \tag{11}$$

where the scalar Green's function $d(|\mathbf{r}-\mathbf{r}'|,\omega)$ is given by

$$d(|\mathbf{r}-\mathbf{r}'|,\omega)=-\frac{1}{4\pi\epsilon_0}\left[\theta(\omega)\frac{e^{i\omega|\mathbf{r}-\mathbf{r}'|/c}}{|\mathbf{r}-\mathbf{r}'|}+\theta(-\omega)\frac{e^{-i\omega|\mathbf{r}-\mathbf{r}'|/c}}{|\mathbf{r}-\mathbf{r}'|}\right]. \tag{12}$$

Thus, the causal Green's function depends only on $|\omega|$, and it is translation invariant as expected for a free-field propagator. Since the **E** field propagator is the only electromagnetic propagator used in this work, we shall omit the superscript E and write simply D_{ij} from now on.

C. The oscillator propagators

The system and bath oscillator Hamiltonian H_{O-R} consists of three parts,

$$H_{O-R}=H'_O+H'_R+H'_{OR}.$$

There is the free (though frequency shifted) system oscillator

$$H'_O=\int d^3\mathbf{r}\left(\frac{P^2}{2\mathcal{M}}+\frac{1}{2}\mathcal{M}\omega_1^2X^2\right), \tag{13}$$

with

$$\omega_1^2=\omega_0^2+\frac{1}{\mathcal{M}}\int_0^\infty d\nu\rho_\nu\nu^2. \tag{14}$$

The free reservoir is described by

$$H'_R=\int d^3\mathbf{r}\int_0^\infty d\nu\left(\frac{1}{2\rho_\nu}Z_\nu^2+\frac{1}{2}\rho_\nu\nu^2Y_\nu^2\right). \tag{15}$$

The coupling between the system and the reservoir oscillators is

$$H'_{OR}=-\int d^3\mathbf{r}\int_0^\infty d\nu\rho_\nu\nu^2\mathbf{X}\cdot\mathbf{Y}_\nu. \tag{16}$$

We shall deal with the coupling in Sec. III B, but for the moment we are going to determine the Green's functions just for the free system and reservoir oscillators.

The free system-oscillator Green's function is defined by

$$K^{(0)}_{ij}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{\hbar}\langle 0|\mathsf{T}(X_i(\mathbf{r},t)X_j(\mathbf{r}',t'))|0\rangle.$$

It is easily calculated from the free-oscillator Hamiltonian (13) as

$$K^{(0)}_{ij}(\mathbf{r},\mathbf{r}',t,t')=-\frac{i}{2\mathcal{M}\omega_1}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')[\theta(t-t')e^{-i\omega_1(t-t')}+\theta(t'-t)e^{i\omega_1(t-t')}]. \tag{17}$$

Fourier transformation with respect to $t-t'$ yields

$$K^{(0)}_{ij}(\mathbf{r},\mathbf{r}';\omega)=\frac{1}{\mathcal{M}}\frac{1}{\omega^2-\omega_1^2+i\varepsilon}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\equiv K^{(0)}(\omega)\,\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \tag{18}$$

The reservoir Green's function is defined by

$$H^{(0)}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu,\nu')=-\frac{i}{\hbar}\langle 0|\mathsf{T}(Y_i(\mathbf{r},t,\nu)Y_j(\mathbf{r}',t',\nu'))|0\rangle.$$

As seen from Eq. (15), the reservoir is simply a collection of mutually independent oscillators, and thus we can write down its Green's function by just copying the structure of Eq. (17),

$$H^{(0)}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu,\nu')=-\frac{i}{2\rho_\nu\nu}\delta_{ij}\delta(\nu-\nu')[\theta(t-t')\times e^{-i\nu(t-t')}+\theta(t'-t)e^{i\nu(t-t')}].$$

Fourier transforming with respect to $t-t'$ and naming the Fourier variable ω, we obtain

$$H^{(0)}_{ij}(\mathbf{r},\mathbf{r}',\nu,\nu';\omega)=H^{(0)}(\omega,\nu)\,\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\,\delta(\nu-\nu'), \tag{19}$$

with

$$H^{(0)}(\omega,\nu)=\frac{1}{\rho_\nu}\frac{1}{\omega^2-\nu^2+i\varepsilon}. \tag{20}$$

III. PERTURBATIVE EXPANSION FOR THE ATOMIC PROPAGATOR

Our aim is to calculate the propagator of the atomic electron when the atom is interacting with the electromagnetic field and via that with the small body and the reservoir. The poles of the full atomic propagator will yield the interaction energy of the atom with the dielectric and the corresponding

radiative lifetime. The general expression for the perturbative expansion of a Green's function is [15,16]

$$G(x,x',t,t')=\sum_{n=0}^{\infty}\left(\frac{-i}{\hbar}\right)^{n+1}\int dt_1\cdots\int dt_n\langle\Omega|\mathsf{T}(\Phi(x,t)\times\Phi^{\dagger}(x',t')V(t_1)\cdots V(t_n))|\Omega\rangle_{conn}.$$

The subscript "*conn*" means that the summation runs only over terms that correspond to connected Feynman diagrams. Topologically equivalent diagrams, i.e., those that can be generated from each other by permuting the factors of $V(t_i)$ in the above expression, are counted only once, and therefore we have omitted the factor $1/n!$ that would have arisen in the straightforward expansion of the time-ordered exponential.

In order to obtain the atomic-electron Green's function, we replace the general Φ fields by the atomic operators c_l, $c_l^{\dagger}$,

$$G_{ii}(t,t')=\sum_{n=0}^{\infty}\left(\frac{-i}{\hbar}\right)^{n+1}\int dt_1\int dt_2\cdots\int dt_n\langle\Omega|\mathsf{T}(c_i(t)\times c_i^{\dagger}(t')V(t_1)V(t_2)\cdots V(t_n))|\Omega\rangle_{conn}. \quad (21)$$

According to Eq. (4), the interaction Hamiltonian in the interaction picture reads

$$V(t)=\int d^3\mathbf{r}\left[e\sum_{ij}\mathbf{q}_{ij}c_i^{\dagger}(t)c_j(t)\delta(\mathbf{r}-\mathbf{R})\mathbf{E}(\mathbf{r},t)-g(\mathbf{r})\mathbf{X}(\mathbf{r},t)\cdot\mathbf{E}(\mathbf{r},t)\right]. \quad (22)$$

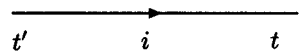
However, in order to account for the interaction between the system and the bath oscillators, we also need to include the coupling Hamiltonian H'_{OR} of Eq. (16). In the interaction picture, it is

$$H'_{OR}(t)=-\int d^3\mathbf{r}\int_0^{\infty}d\nu\rho_{\nu}\nu^2\mathbf{X}(\mathbf{r},t)\cdot\mathbf{Y}_{\nu}(\mathbf{r},t). \quad (23)$$

Thus, the interaction Hamiltonian $V(t)$ that appears in Eq. (21) should *prima facie* be the sum of $V(t)$, Eq. (22), and $H'_{OR}(t)$, Eq. (23). However, as we shall see below, we can take into account the interaction between the system and the bath oscillators to all orders by dressing the system-oscillator field. This will allow us to omit $H'_{OR}(t)$ from the interaction Hamiltonian while using the dressed-system oscillator field in $V(t)$, Eq. (22).

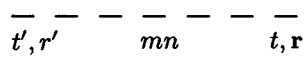
A. Feynman rules

The terms appearing in the perturbation expansion (21) can be represented graphically by Feynman diagrams. We use the following lines for the various free propagators:

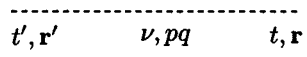


corresponds to $i\hbar G_{ii}^{(0)}(t,t')$,

corresponds to $i\hbar D_{kl}(\mathbf{r},t;\mathbf{r}',t')$,

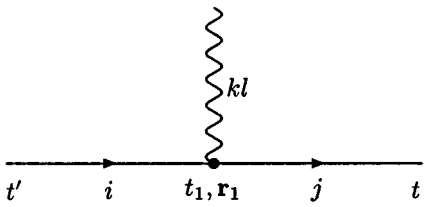


corresponds to $i\hbar K_{mn}^{(0)}(\mathbf{r},t;\mathbf{r}',t')$,

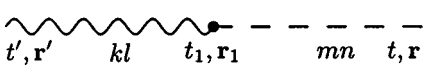


corresponds to $i\hbar H_{pq}^{(0)}(\mathbf{r},t;\mathbf{r}',t';\nu,\nu')$.

The interaction Hamiltonian of Eq. (22) yields the following rules for the vertices between the atomic electron and the photon field and between the photon field and the system oscillator:

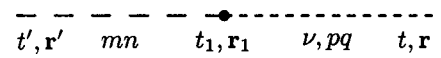


represents $eq_{ij}^{(k)}\delta(\mathbf{r}_1-\mathbf{R})$,



represents $-\delta_{lm}g(\mathbf{r}_1)$.

(The symbol $q_{ij}^{(k)}$ denotes the kth vector component of the transition amplitude $\mathbf{q}_{ij}$ between the atomic states i and j.) The coupling Hamiltonian of Eq. (23) gives the vertex between the system and the bath oscillator fields:



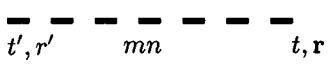
represents $-\rho_\nu \nu^2 \delta_{np}$.

To compute a diagram, one has to sum over all internal indices and integrate over internal times, internal coordinates, and reservoir oscillator frequencies ν and ν'.

B. Dressing the system-oscillator line

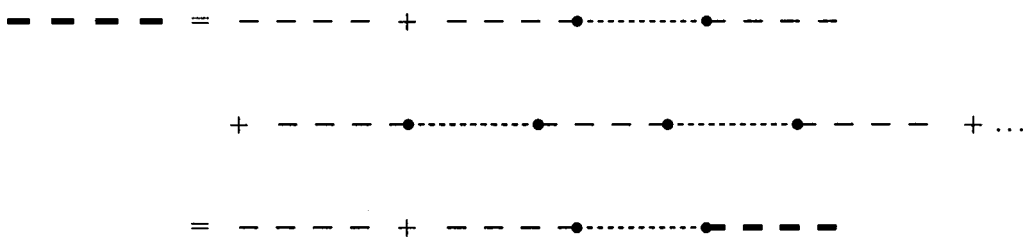
The atom interacts with the system oscillator via the electromagnetic field, and we treat this interaction perturbatively. However, the interaction of the system oscillator with the bath can be summed to all orders, i.e., the corrections to the system-oscillator dynamics due to the presence of the reservoir can be treated exactly. To do this we perform quite a simple summation of all relevant graphs, which is going to result in an easily soluble Dyson equation. In fact, this summation corresponds to the diagonalization of the polarization Hamiltonian performed by Huttner and Barnett [5].

We choose a bold dashed line to represent the dressed system oscillator, i.e.,



stands for $i\hbar K_{mn}(\mathbf{r},t;\mathbf{r}',t')$.

Then, we have the following graphical equation:



which corresponds to

$$K_{mn}(\mathbf{r},t;\mathbf{r}',t') = K_{mn}^{(0)}(\mathbf{r},t;\mathbf{r}',t') + \sum_{l,p} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int d^3r_1 \times \int d^3r_2 \int_0^{\infty} d\nu \int_0^{\infty} d\nu' \nu^2 \nu'^2 \rho_\nu \rho_{\nu'} \times K_{ml}^{(0)}(\mathbf{r},t;\mathbf{r}_1,t_1) H_{lp}^{(0)}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\nu,\nu') \times K_{pn}(\mathbf{r}_2,t_2;\mathbf{r}',t').$$

Fourier transforming both sides of the above equation with respect to time and using Eqs. (18) and (19), we obtain

$$K_{mn}(\mathbf{r},\mathbf{r}';\omega) = K^{(0)}(\omega)\delta_{mn}\delta(\mathbf{r}-\mathbf{r}') + \int_0^{\infty} d\nu \rho_\nu^2 \nu^4 K^{(0)}(\omega) H^{(0)}(\omega,\nu) K_{mn}(\mathbf{r},\mathbf{r}',\omega).$$

Thus, we find for the dressed system-oscillator propagator

$$K_{mn}(\mathbf{r},\mathbf{r}';\omega) = K(\omega)\delta_{mn}\delta(\mathbf{r}-\mathbf{r}') \quad \text{with} \quad K(\omega) = \left[\frac{1}{K^{(0)}(\omega)} - \int_0^{\infty} d\nu \rho_\nu^2 \nu^4 H^{(0)}(\omega,\nu) \right]^{-1}.$$

Substituting from Eqs. (18), (20), and (14), we get

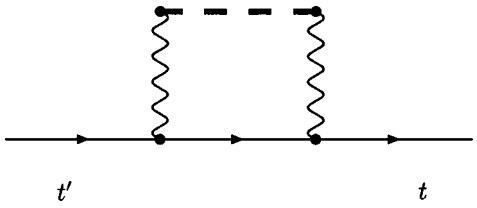
$$K(\omega) = \left[\mathcal{M}(\omega^2 - \omega_0^2 + i\varepsilon) - \omega^2 \int_0^{\infty} d\nu \frac{\rho_\nu \nu^2}{\omega^2 - \nu^2 + i\varepsilon} \right]^{-1}. \tag{24}$$

C. Atomic propagator to the fourth order

Looking at the possible ways of combining the lines and vertices of Sec. III A, one quickly sees that the first nonvanishing order to which the atom-oscillator interactions contribute to the exact atomic propagator is the fourth order. To the second order, both the atom and the oscillator can interact only with the electromagnetic field, but not with each other. The second-order terms yield part of the self-energies of atom and oscillator, but these are of no concern to us for the purpose of the present paper. They do have physical meaning

as the difference between the self-energies of a bound electron and of a free electron accounts for most of the Lamb shift, but we note that even if we were interested in them, the present model would not be suitable for calculating Lamb shifts because in our interaction Hamiltonian we have neglected terms that contain the square of the polarization. In the following, we shall assume that the energies E_l which enter the atomic-electron propagators are not bare but already renormalized and have already been corrected for any Lamb shift contributions.

Hence, the first nonzero corrections to the atomic energies due to atom-oscillator interactions appear in the fourth order of the perturbation expansion. We choose the symbol $G_{ii}^{(4)}(t,t')$ to denote the fourth-order correction to the Green's function that is due only to the interaction with the oscillator. The only graph that contributes to the electromagnetically mediated interaction between the atom and the oscillator is



Note, in particular, that the same graph but with crossing photon lines is, in fact, not different since the oscillator line is not directed, $K(t_1-t_2)=K(t_2-t_1)$. Using the Feynman rules of Sec. III A and taking advantage of the structure of the propagators, $K_{lm}(\mathbf{r},\mathbf{r}',t,t')=K(t-t')\delta_{lm}\delta(\mathbf{r}-\mathbf{r}')$, $G_{ij}(t,t')=G_{ii}(t-t')\delta_{ij}$, one obtains for the contribution of the above graph

$$G_{ii}^{(4)}(t,t')=i\hbar e^2\sum_j\sum_{l,m,n}\int_{-\infty}^{\infty}dt_1\cdots\int_{-\infty}^{\infty}dt_4 q_{ij}^{(m)}q_{ji}^{(n)}\times K(t_1-t_2)\int d^3\mathbf{r}g^2(\mathbf{r})D_{lm}(\mathbf{r},\mathbf{R},t_1-t_3)\times D_{ln}(\mathbf{r},\mathbf{R},t_2-t_4)G_{ii}^{(0)}(t-t_3)G_{jj}^{(0)}\times(t_3-t_4)G_{ii}^{(0)}(t_4-t'). \quad (25)$$

Up to this point, all our considerations have been perfectly general and suitable for the description of arbitrary dielectric bodies. Now we specify that our dielectric is actually a point at the coordinate origin by taking the coupling between the system oscillator and the electromagnetic field to be

$$g^2(\mathbf{r})=V_0\delta(\mathbf{r}), \quad (26)$$

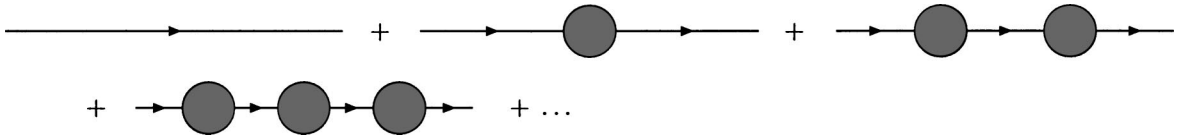
where V_0 is the volume of the small dielectric body. Then, the spatial integration in Eq. (25) is straightforward to perform. Thus, we find that the Fourier-transformed atomic-electron Green's function can be written as

$$G_{ii}(E)\approx G_{ii}^{(0)}(E)+G_{ii}^{(4)}(E), \quad (27)$$

where the Fourier-transformed fourth-order correction $G_{ii}^{(4)}(E)$ is given by

$$G_{ii}^{(4)}(E)=\frac{i\hbar e^2V_0}{2\pi}\sum_j\sum_{l,m,n}q_{ij}^{(\mu)}q_{ji}^{(\nu)}\int_{-\infty}^{\infty}d\omega K(\omega)D_{lm}\times(0,\mathbf{R},-\omega)D_{ln}(0,\mathbf{R},\omega)G_{ii}^{(0)}(E)G_{jj}^{(0)}\times(E-\hbar\omega)G_{ii}^{(0)}(E).$$

However, since we are not actually interested in corrections to the Green's function itself but rather in the perturbative shifts of its poles, Eq. (27) is not yet quite what we need. Instead, we consider the Green's function that corresponds to the following sum of diagrams:



Here, every blob represents the fourth-order approximation to the atomic-electron self-energy shown in the previous diagram, but without the two external lines. The Fourier transform of this blob equals

$$\Sigma_i(E)=\frac{i\hbar e^2V_0}{2\pi}\sum_j\sum_{l,m,n}q_{ij}^{(m)}q_{ji}^{(n)}\int_{-\infty}^{\infty}d\omega K(\omega)\times D_{lm}(0,\mathbf{R},\omega)D_{ln}(0,\mathbf{R},\omega)G_{jj}^{(0)}(E-\hbar\omega), \quad (28)$$

where we have used the fact that $D(\mathbf{r},\mathbf{r}',-\omega)=D(\mathbf{r},\mathbf{r}',\omega)$. The summation of this series of diagrams is quite simple and gives

$$G_{ii}(E)=\frac{G_{ii}^{(0)}}{1-G_{ii}^{(0)}(E)\Sigma_i(E)}.$$

Substituting the explicit form of $G_{ii}^{(0)}(E)$ from Eq. (8), one obtains

$$G_{ii}(E)=\frac{1}{E-E_i+i\varepsilon-\Sigma_i(E)}. \quad (29)$$

Since the diagram with two blobs is of the eighth order and the following ones are of even higher orders, the above

Green's function (29) is in fact the same as the one in Eq. (27) to the order we are considering, namely, the fourth order. However, the poles of Eq. (29) are much easier to locate than those of Eq. (27). The poles of the perturbatively corrected causal Green's function yield the atomic energies as shifted due to the presence of the oscillator. To obtain them explicitly is still a formidable task—since Σ_i is a function of E one has to solve a complicated integral equation to find the poles of $G_{ii}(E)$. One can, however, look for the approximate location of a pole by approximating $\Sigma_i(E)$ by $\Sigma_i(E_i)$ in Eq. (29). Thus, the poles of the Green's function $G_{ii}(E)$ are located approximately at

$$E = E^* \approx E_i + \Sigma_i(E_i), \tag{30}$$

and the approximate shift of the ith energy level is $\Sigma_i(E_i)$.

We would like to point out here that the propagator $G_{ii}(E)$ of the interacting system has also other poles that are not close to the location of the poles of the noninteracting Green's function $G_{ii}^{(0)}(E)$. These poles tend to lie far into the complex plane, and thus correspond to rapidly decaying modes. For the dynamics of the system, all poles of the propagator are important, and for the problem of two oscillators interacting via the electromagnetic field it has been shown that the additional poles far into the complex plane do, in fact, dominate the short-time dynamics of the system [21]. In the present context, however, we are not interested in the dynamics of the system but just in the stationary energy shift, and hence we ignore those additional poles in the complex plane.

In the following section, we shall analyze the energy shifts $\Sigma_i(E_i)$ in some detail. In order to extract readily usable information on the physics of our system, we shall consider two important asymptotic regimes. In the nonretarded regime or ''near zone,'' the distance R of the atom from the oscillator is small compared to both the wavelength of any atomic transition that contributes significantly to the self-energy and the wavelength of the oscillator. This is to say that both dimensionless combinations $|E_i - E_j|R/(\hbar c)$ and $\omega_0 R/c$ are small. In the retarded regime or ''far zone,'' the opposite holds true and $|E_i - E_j|R/(\hbar c)$ and $\omega_0 R/c$ are both large.

IV. ATOM-OSCILLATOR INTERACTION ENERGY AND DECAY RATES

Substituting the explicit form of the electromagnetic causal Green's function, Eqs. (11) and (12), into the self-energy, Eq. (28), and canceling the antisymmetric part of the integrand under the symmetric ω integral, we can write the self-energy as

$$\Sigma_i(E_i) = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{ie^2 V_0}{\pi} \sum_j \sum_{l,m,n} \omega_{ij} q_{ij}^{(m)} q_{ji}^{(n)} \int_0^\infty d\omega \times \left[\mathcal{M}(\omega^2 - \omega_0^2 + i\varepsilon) - \omega^2 \int_0^\infty d\nu \frac{\rho_\nu \nu^2}{\omega^2 - \nu^2 + i\varepsilon}\right]^{-1} \times \left[(-\nabla^2 \delta_{lm} + \nabla_l \nabla_m)\frac{e^{ikR}}{R}\right] \times \left[(-\nabla^2 \delta_{ln} + \nabla_l \nabla_n)\frac{e^{ikR}}{R}\right] \frac{1}{(\omega_{ij} + i\varepsilon)^2 - \omega^2}, \tag{31}$$

where $R = |\mathbf{R}|$, $k = \omega/c$, $\omega_{ij} = (E_i - E_j)/\hbar$.

At this point we would like to consider several distinct cases.

A. Atom in the ground state interacting with an undamped oscillator

For studying the interaction of the atom with an undamped oscillator, we just need to set to zero the coupling of the oscillator to the reservoir. Then, the Green's function (24) for the system oscillator reduces to

$$K(\omega) = \frac{1}{\mathcal{M}} \frac{1}{\omega^2 - \omega_0^2 + i\varepsilon}.$$

Correspondingly, we set $\rho_\nu = 0$ in Eq. (31), which simplifies the expression for $\Sigma_i(E_i)$ considerably. Further simplification is brought about if the atomic state i we are considering is the ground state. Then, we have $\omega_{0j} < 0$ for all other states j, and thus the only poles of the integrand in Eq. (31) are at $-\omega_0 + i\varepsilon$, $\omega_0 - i\varepsilon$, $-|\omega_{0j}| + i\varepsilon$, and $|\omega_{0j}| - i\varepsilon$. This means that there are no poles in the first quadrant of the complex ω plane, and we can rotate the ω integration by $\pi/2$ to run along the positive imaginary axis from 0 to $i\infty$. Substituting $i = 0$ for the ground state and changing variables from ω to $i\xi$ we can rewrite the self-energy as

$$\Sigma_0(E_0) = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\pi\mathcal{M}} \sum_{j>0} \sum_{l,m,n} q_{0j}^{(m)} q_{j0}^{(n)} \omega_{j0} \times \int_0^\infty \frac{d\xi}{\xi^2 + \omega_0^2} \left[(-\delta_{lm}\xi^2/c^2 + \nabla_l \nabla_m)\frac{e^{-\xi R/c}}{R}\right] \times \left[(-\delta_{ln}\xi^2/c^2 + \nabla_l \nabla_n)\frac{e^{-\xi R/c}}{R}\right] \frac{1}{\xi^2 + \omega_{j0}^2}. \tag{32}$$

This result is well known and has been derived many times (cf., e.g., Refs. [22,23]). We have rederived it in a different way for the sake of comparison with the case of a damped oscillator. Carrying out the differentiations, we obtain

$$\Sigma_0(E_0) = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\pi\mathcal{M}} \sum_{j>0} \omega_{j0} \int_0^\infty d\xi e^{-2\xi R/c} \frac{1}{\xi^2 + \omega_0^2} \times \frac{1}{\xi^2 + \omega_{j0}^2} \left[|\mathbf{q}_{0j}^{(\perp)}|^2 \left(\frac{(\xi/c)^2}{R} + \frac{\xi/c}{R^2} + \frac{1}{R^3}\right)^2 + 4|q_{0j}^{(\parallel)}|^2 \left(\frac{\xi/c}{R^2} + \frac{1}{R^3}\right)^2\right], \tag{33}$$

where we have split the vector $\mathbf{q}_{ij}$ into its component $q_{ij}^{(\parallel)}$ along $\mathbf{R}$ and the two-component vector $\mathbf{q}_{ij}^{(\perp)}$ perpendicular to $\mathbf{R}$.

In the nonretarded regime when $\omega_{j0}R/c$ and $\omega_0 R/c$ are both small, the R^{-3} terms dominate and the exponential can be approximated by $\exp(-2\xi R/c)\approx 1$. Then, performing the integration over ξ, one finds the following ground-state energy shift:

$$\Delta E_0^{(\mathrm{nonret})} \approx -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{2\mathcal{M}\omega_0}\frac{1}{R^6}\sum_{j>0}\frac{1}{\omega_0+\omega_{j0}}(|\mathbf{q}_{0j}^{(\perp)}|^2 + 4|q_{0j}^{(\parallel)}|^2). \tag{34}$$

In the retarded regime when $\omega_{j0}R/c$ and $\omega_0 R/c$ are both large, the analysis is slightly more involved. We first rewrite Eq. (33) by expressing the polynomial in ξ in terms of derivatives of the exponential,

$$\Delta E_0 = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\pi\mathcal{M}}\sum_{j>0}\omega_{j0}\left(\frac{|\mathbf{q}_{0j}^{(\perp)}|^2}{16R^2}\frac{\partial^4}{\partial R^4} - \frac{|\mathbf{q}_{0j}^{(\perp)}|^2}{4R^3}\frac{\partial^3}{\partial R^3} + \frac{3|\mathbf{q}_{0j}^{(\perp)}|^2+4|q_{0j}^{(\parallel)}|^2}{4R^4}\frac{\partial^2}{\partial R^2} - \frac{|\mathbf{q}_{0j}^{(\perp)}|^2+4|q_{0j}^{(\parallel)}|^2}{R^5}\frac{\partial}{\partial R} + \frac{|\mathbf{q}_{0j}^{(\perp)}|^2+4|q_{0j}^{(\parallel)}|^2}{R^6}\right)\int_0^\infty d\xi \frac{\exp(-2\xi R/c)}{(\xi^2+\omega_0^2)(\xi^2+\omega_{j0}^2)}. \tag{35}$$

The ξ integral equals [24] [formula (5.2.12)]

$$\int_0^\infty d\xi \frac{\exp(-2\xi R/c)}{(\xi^2+\omega_0^2)(\xi^2+\omega_{j0}^2)} = \frac{1}{\omega_{j0}^2-\omega_0^2}\left[\frac{1}{\omega_0}f(2\omega_0 R/c) - \frac{1}{\omega_{j0}}f(2\omega_{j0}R/c)\right].$$

Note that the limit $\omega_{j0}=\omega_0$ is innocuous, since the square bracket vanishes linearly with $\omega_{j0}-\omega_0$ and, consequently, nothing special happens when one of the atomic transitions coincides with the oscillator frequency. Using the asymptotic expansion of the function $f(z)=\mathrm{ci}(z)\sin(z)-\mathrm{si}(z)\cos(z)$ for large z [24] [formula (5.2.34)]

$$f(z)\approx \frac{1}{z}-\frac{2!}{z^3}+\frac{4!}{z^5}-\cdots,$$

we find that the ground-state energy shift in the retarded regime is approximately

$$\Delta E_0^{(\mathrm{ret})} \approx -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 c V_0}{4\pi\mathcal{M}\omega_0^2}\frac{1}{R^7}\sum_{j>0}\frac{1}{\omega_{j0}}(13|\mathbf{q}_{0j}^{(\perp)}|^2 + 20|q_{0j}^{(\parallel)}|^2) + O\left(\frac{1}{R^9}\right). \tag{36}$$

This is the Casimir-Polder result for the retarded interaction between two neutral, polarizable point particles [25].

B. Atom in the ground state interacting with a damped oscillator

We now want to evaluate the self-energy (31) for the case of a damped system oscillator when the bath oscillator masses ρ_ν are not zero. To be able to proceed we need to specify the dependence of ρ_ν on the frequency ν, which can be done by comparing our model with experimentally observable characteristics of absorbing dielectrics. The polarization of the system oscillator is a response to the electric field,

$$\langle P_i(\mathbf{r},t)\rangle = \int_0^\infty d\tau \int d^3\mathbf{r}'\,\alpha_{ij}(\mathbf{r},\mathbf{r}',\tau)\langle E_j(\mathbf{r}',t-\tau)\rangle,$$

and the response function $\alpha_{ij}(\mathbf{r},\mathbf{r}',\tau)$ can very simply be calculated by linear-response theory, i.e., by first-order perturbation theory. For the oscillator-field interaction given in Eq. (4), we obtain

$$\alpha_{ij}(\mathbf{r},\mathbf{r}',\tau) = \theta(\tau)\frac{i}{\hbar}g(\mathbf{r})g(\mathbf{r}')\langle 0|[X_i(\mathbf{r},t),X_j(\mathbf{r}',t-\tau)]|0\rangle \equiv -g(\mathbf{r})g(\mathbf{r}')K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';t,t-\tau).$$

Here, $K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';t,t')$ is the retarded propagator of the dressed system-oscillator field, which we calculate in the Appendix from the equations of motion for the oscillator fields.

The experimentally significant quantity is the polarization $\alpha_{ij}(\mathbf{r},\mathbf{r}',\omega)$ which is the Fourier transform of the response function. We obviously have $\alpha_{ij}(\mathbf{r},\mathbf{r}',\omega)=\alpha(\mathbf{r},\omega)\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')$, i.e., our model for the dielectric is isotropic and precludes spatial dispersion. From the Fourier transform of the retarded system-oscillator propagator, Eq. (A4), derived in the Appendix, we find

$$\alpha_{ij}(\mathbf{r},\mathbf{r}';\omega) = -g^2(\mathbf{r})\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\left[\mathcal{M}(\omega^2-\omega_0^2) - \omega^2\int_0^\infty d\nu \frac{\rho_\nu \nu^2}{\omega^2-\nu^2+i\varepsilon\omega}\right]^{-1}. \tag{37}$$

For the response function $\alpha_{ij}(\mathbf{r},\mathbf{r}';\tau)$ to satisfy causality requirements, we must ensure that its Fourier transform $\alpha_{ij}(\mathbf{r},\mathbf{r}';\omega)$ has no poles in the upper half of the complex ω plane. We show in the Appendix that this is indeed the case, provided the function ρ_ν for real ν is real, even, and positive and does not have poles on the real axis.

The ν integral in Eqs. (24) and (37) does not converge at the upper limit unless ρ_ν falls off as ν^{-2} or faster for large ν. If we introduce a new constant γ and choose

$$\rho_\nu = \frac{4\mathcal{M}\gamma}{\pi\nu^2}, \tag{38}$$

we obtain for the polarizability [26]

$$\alpha(\omega) = \frac{1}{\mathcal{M}}\frac{1}{\omega_0^2-\omega^2-2i\gamma\omega},$$

where we have got rid of the **r** dependence by using the explicit expression for $g^2(\mathbf{r})$, Eq. (26), and averaging over the volume of the small dielectric body. Thus, the choice of ρ_ν, Eq. (38), leads to a single absorption line with Lorentzian shape and a width that is given by the damping constant γ. Different choices for ρ_ν can be used not just to model different line shapes but also to introduce additional poles in $\alpha(\omega)$ and thus describe more than one absorption line, as has already been pointed out for the canonically quantized version of this model [5]. Such an approach is certainly well suited to describing broad bands of absorption, and experimentally observed absorption profiles can even be matched by fitting them to Eq. (37). However, when it comes to modeling several widely spaced absoption lines, one might more appropriately choose a model that right from the outset has several system-oscillator frequencies, because this is likely to be the technically simpler choice and would make the physics of the processes involved more transparent.

Having specified ρ_ν we can proceed with the evaluation of the self-energy. Carrying out the ν integral in Eq. (31) we see that the resulting expression does not have any poles again in the first quadrant of the complex ω plane. Thus, we rotate the contour of the ω integration by $\pi/2$ and change variables from ω to $i\xi$, as before, and obtain

$$\Sigma_0(E_0)=-\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{e^2V_0}{\pi\mathcal{M}}\sum_{j>0}\sum_{l,m,n}q_{0j}^{(m)}q_{j0}^{(n)}\omega_{j0}\times\int_0^\infty d\xi\frac{1}{\xi^2+\omega_0^2+2\xi\gamma}\times\left[(-\delta_{lm}\xi^2/c^2+\nabla_l\nabla_m)\frac{e^{-\xi R/c}}{R}\right]\times\left[(-\delta_{ln}\xi^2/c^2+\nabla_l\nabla_n)\frac{e^{-\xi R/c}}{R}\right]\frac{1}{\xi^2+\omega_{j0}^2}. \quad (39)$$

The only difference between this expression and the equivalent for the undamped case in Eq. (32) is the presence of $2\xi\gamma$ in the denominator of the first factor. This makes the ξ integral a little more complicated than before, but the calculation runs along the same lines as for the undamped case. First, we carry out the differentiations and get the same result as in Eq. (33), except for the replacement of the first denominator $\xi^2+\omega_0^2$ by $\xi^2+\omega_0^2+2\xi\gamma$.

In the nonretarded regime where the distance R between the atom and the oscillator is small compared to all relevant wavelengths, we can make the same approximations as before and obtain for the ground-state energy shift

$$\Delta E_0^{(\mathrm{nonret})}\approx-\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{e^2V_0}{2\mathcal{M}\omega_0}\frac{1}{R^6}\sum_{j>0}\times\frac{\omega_{j0}\omega_0\mathcal{J}(\omega_{j0},\omega_0,\gamma)}{(\omega_{j0}^2-\omega_0^2)^2+4\gamma^2\omega_{j0}^2}(|\mathbf{q}_{0j}^{(\perp)}|^2+4|q_{0j}^{(\parallel)}|^2). \quad (40)$$

The fraction after the sum sign stems from the integral over ξ and replaces $(\omega_0+\omega_{j0})^{-1}$ in the result for the undamped case, Eq. (34). The abbreviation $\mathcal{J}(\omega_{j0},\omega_0,\gamma)$ stands for

$$\mathcal{J}(\omega_{j0},\omega_0,\gamma)=\frac{4\gamma}{\pi}\ln\left(\frac{\omega_{j0}}{\omega_0}\right)-\frac{\omega_{j0}^2-\omega_0^2}{\omega_{j0}}+\begin{cases}\dfrac{\omega_{j0}^2-\omega_0^2+2\gamma^2}{\sqrt{\omega_0^2-\gamma^2}}\left(1-\dfrac{2}{\pi}\arctan\dfrac{\gamma}{\sqrt{\omega_0^2-\gamma^2}}\right) & \gamma<\omega_0,\\ 2\dfrac{\omega_{j0}^2+\gamma^2}{\pi\gamma} & \gamma=\omega_0,\\ \dfrac{\omega_{j0}^2-\omega_0^2+2\gamma^2}{\pi\sqrt{\gamma^2-\omega_0^2}}\ln\left(\dfrac{\gamma+\sqrt{\gamma^2-\omega_0^2}}{\gamma-\sqrt{\gamma^2-\omega_0^2}}\right) & \gamma>\omega_0.\end{cases}$$

Taking the limit of no damping, $\gamma\to0$, we recover Eq. (34), as we should.

We can go beyond the nonretarded approximation by re-expressing the energy-level shift in the same way as in Eq. (35), except for the replacement of the factor $(\xi^2+\omega_0^2)^{-1}$ by $(\xi^2+\omega_0^2+2\gamma\xi)^{-1}$ under the ξ integral. The ξ integration can then still be performed analytically—one gets a combination of terms involving exponential integrals. While this would be useful for the evaluation of the level shift for arbitrary distances R, one can take a simpler approach for determining the shift in the retarded limit where R is much larger than any relevant wavelength. When R is large, the exponential strongly damps the integrand for increasing ξ. Provided that the rest of the integrand is smooth, it can be Taylor expanded around the lower limit of the integral and integrated with the exponential to give an asymptotic approximation to the integral (Watson's Lemma, see, e.g., Ref. [27]), i.e.,

$$\int_0^\infty d\xi\frac{\exp(-2\xi R/c)}{(\xi^2+\omega_0^2+2\gamma\xi)(\xi^2+\omega_{j0}^2)}\approx\int_0^\infty d\xi\exp(-2\xi R/c)\times\left(\frac{1}{\omega_0^2\omega_{j0}^2}-\frac{2\gamma}{\omega_0^4\omega_{j0}^2}\xi+O(\xi^2)\right).$$

In this way, we obtain for the level shift in the retarded regime

$$\Delta E_0^{(\mathrm{ret})} \approx -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 c V_0}{4\pi \mathcal{M}\omega_0^2} \sum_{j>0} \frac{1}{\omega_{j0}} \times \left(\frac{13|\mathbf{q}_{0j}^{(\perp)}|^2 + 20|q_{0j}^{(\parallel)}|^2}{R^7} - \frac{6\gamma}{\omega_0^2} \frac{7|\mathbf{q}_{0j}^{(\perp)}|^2 + 6|q_{0j}^{(\parallel)}|^2}{R^8} \right) + O\left(\frac{1}{R^9}\right). \tag{41}$$

Thus, we find that in the retarded regime or far zone damping makes no difference to the leading order. It does, however, give rise to a next-to-leading-order correction proportional to R^{-8} and growing linearly with the damping parameter γ. As seen by comparison with Eq. (36), such R^{-8} terms were not present in the undamped case when the next-to-leading-order terms were only of the order of R^{-9}. We conclude that damping has more impact at small and intermediate distances R between the atom and the oscillator than at large R.

C. Atom in an excited state interacting with a damped oscillator

If the atom is not in the ground state but in an excited state, the evaluation of the self-energy is more complicated. With our choice of ρ_ν, Eq. (38), we get from Eq. (31),

$$\Sigma_i(E_i) = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{ie^2 V_0}{\pi\mathcal{M}} \sum_j \sum_{l,m,n} \omega_{ij} q_{ij}^{(m)} q_{ji}^{(n)} \times \int_0^\infty d\omega \frac{1}{\omega^2 - \omega_0^2 + i\varepsilon + 2i\gamma\omega} \times \left[(-\nabla^2 \delta_{lm} + \nabla_l \nabla_m) \frac{e^{i\omega R/c}}{R} \right] \times \left[(-\nabla^2 \delta_{ln} + \nabla_l \nabla_n) \frac{e^{i\omega R/c}}{R} \right] \frac{1}{(\omega_{ij} + i\varepsilon)^2 - \omega^2}.$$

Now the integrand has poles in the first quadrant of the ω plane, namely, at $\omega_{ij} + i\varepsilon$ for all states $j < i$. Thus, if we rotate the contour by $\pi/2$, we get an integral along the positive imaginary axis $\omega = i\xi$ and a sum of residues from these poles,

$$\Sigma_i(E_i) = \Sigma_i^{(\mathrm{int})}(E_i) + \Sigma_i^{(\mathrm{res})}(E_i).$$

The integral along the positive imaginary axis leads to identical expressions as in Sec. IV B, except for the replacement of ω_{j0} by ω_{ji} and of $\mathbf{q}_{0j}$ by $\mathbf{q}_{ij}$. Apart from these substitutions, $\Sigma_i^{(\mathrm{int})}(E_i)$ is the same as $\Sigma_0(E_0)$ in Eq. (39), and consequently this part of the self-energy gives rise to energy shifts that are otherwise identical to Eqs. (40) and (41) in the nonretarded and retarded regimes, respectively. Thus, we shall now concentrate on analyzing the part of the self-energy that arises from the residue contributions around the poles at $\omega_{ij} + i\varepsilon$ for $j < i$. We find

$$\Sigma_i^{(\mathrm{res})}(E_i) = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\mathcal{M}} \sum_{j<i} \left(\frac{|\mathbf{q}_{ij}^{(\perp)}|^2}{16R^2} \frac{\partial^4}{\partial R^4} - \frac{|\mathbf{q}_{ij}^{(\perp)}|^2}{4R^3} \frac{\partial^3}{\partial R^3} + \frac{3|\mathbf{q}_{ij}^{(\perp)}|^2 + 4|q_{ij}^{(\parallel)}|^2}{4R^4} \frac{\partial^2}{\partial R^2} - \frac{|\mathbf{q}_{ij}^{(\perp)}|^2 + 4|q_{ij}^{(\parallel)}|^2}{R^5} \frac{\partial}{\partial R} + \frac{|\mathbf{q}_{ij}^{(\perp)}|^2 + 4|q_{ij}^{(\parallel)}|^2}{R^6} \right) \frac{e^{2i\omega_{ij}R/c}}{\omega_{ij}^2 - \omega_0^2 + 2i\gamma\omega_{ij}}. \tag{42}$$

This is a complex quantity. Its real part causes an energy shift

$$\Delta E_i^{(\mathrm{res})} = \mathrm{Re}(\Sigma_i^{(\mathrm{res})}(E_i)),$$

which jointly with the part of the self-energy $\Sigma_i^{(\mathrm{int})}(E_i)$ that results from the integral along the positive imaginary axis, constitutes the total energy shift of the atomic level. The imaginary part of $\Sigma_i^{(\mathrm{res})}(E_i)$ gives rise to a decay rate,

$$\Gamma_i = -\mathrm{Im}[\Sigma_i^{(\mathrm{res})}(E_i)].$$

From Eq. (42) it is easy to extract the asymptotic behavior of these quantities for small and large R. In the nonretarded regime, where $\omega_{ij}R/c \ll 1$, we find for the residue part of the energy shift

$$(\Delta E_i^{(\mathrm{res})})^{(\mathrm{nonret})} \approx \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\mathcal{M}} \sum_{j<i} (|\mathbf{q}_{0j}^{(\perp)}|^2 + 4|q_{0j}^{(\parallel)}|^2) \frac{\omega_{ij}^2 - \omega_0^2}{(\omega_{ij}^2 - \omega_0^2)^2 + 4\gamma^2\omega_{ij}^2} \frac{1}{R^6} + O(R^{-5}), \tag{43}$$

and for the decay rate

$$\Gamma_i^{(\mathrm{nonret})} \approx \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\mathcal{M}} \sum_{j<i} (|\mathbf{q}_{0j}^{(\perp)}|^2 + 4|q_{0j}^{(\parallel)}|^2) \frac{2\gamma\omega_{ij}}{(\omega_{ij}^2 - \omega_0^2)^2 + 4\gamma^2\omega_{ij}^2} \frac{1}{R^6} + O(R^{-5}). \tag{44}$$

In the retarded regime, where $\omega_{ij}R/c \gg 1$, the level shift and the decay rate are approximately

$$(\Delta E_i^{(\mathrm{res})})^{(\mathrm{ret})} \approx \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\mathcal{M}} \sum_{j<i} |\mathbf{q}_{ij}^{(\perp)}|^2 \frac{(\omega_{ij}/c)^4}{R^2} \times \frac{(\omega_{ij}^2 - \omega_0^2)\cos(2\omega_{ij}R/c) + 2\gamma\omega_{ij}\sin(2\omega_{ij}R/c)}{(\omega_{ij}^2 - \omega_0^2)^2 + 4\gamma^2\omega_{ij}^2}, \tag{45}$$

$$\Gamma_i^{(\mathrm{ret})} \approx \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^2 V_0}{\mathcal{M}} \sum_{j<i} |\mathbf{q}_{ij}^{(\perp)}|^2 \frac{(\omega_{ij}/c)^4}{R^2} \times \frac{2\gamma\omega_{ij}\cos(2\omega_{ij}R/c) - (\omega_{ij}^2 - \omega_0^2)\sin(2\omega_{ij}R/c)}{(\omega_{ij}^2 - \omega_0^2)^2 + 4\gamma^2\omega_{ij}^2}. \tag{46}$$

Finally, we would like to point out that the limit of no damping, $\gamma \to 0$, is not admissible in any of the above results. In that limit the residue part of the self-energy in Eq. (42) has an unrecoverable singularity at $\omega_{ij} = \omega_0$, ie the atom can decay into a mode that is in resonance with the oscillator, and resonant transfer occurs. This very special scenario is not covered by the present calculation because for that we would have had to take into account the natural linewidth which arises from the coupling to the electromagnetic field alone. Had we wanted to include it, we would have needed to consider the second-order as well as the fourth-order terms as opposed to just the fourth-order terms in the perturbation expansion. Moreover, the assumption in Eq. (30) that the poles of the exact Green's function are close to those of the unperturbed Green's function is not justified close to resonance.

V. SUMMARY AND CONCLUSIONS

In this paper, we have calculated the energy-level shifts and the decay rates of an atom that interacts with a damped harmonic oscillator a distance R away. Without damping we recover the usual results for the van der Waals interaction between two polarizable, unexcited point particles: the energy shift scales as R^{-6} in the nonretarded regime or near zone where R is small, and as R^{-7} in the retarded regime or far zone where R is large. This is still true when damping is included. Then, however, the coefficient of R^{-6} in the near zone depends on the damping constant γ strongly and nontrivially. In the far zone, damping has no impact on the leading-order R^{-7} term in the interaction energy, but it introduces a new next-to-leading order R^{-8} term, whereas without damping the next-to-leading order is only R^{-9}. If the atom is excited, then a part of the energy shift originates from an integral over photon frequencies and behaves in the same way as for a ground-state atom. The other part is purely due to virtual down transitions in the atom and appears in the calculation as a sum of residues from the integration over photon frequency. Since down transitions can also be real, the residue part of the energy shift goes hand in hand with a modified decay rate of the atom due to the presence of the damped oscillator. In the nonretarded regime, both the energy shift and the decay rate vary with distance as R^{-6} and depend strongly on the damping constant γ. In the retarded regime, the energy shift is dominated by the part that arises from virtual down transitions in the atom. Its distance dependence oscillates with twice the ratio of R to the wavelength of the atomic transition and with an amplitude that drops off as R^{-2}. The decay rate shows qualitatively the same behavior. To leading order the decay takes place preferentially parallel to the atom-oscillator axis, and perpendicular components decay only with a rate that drops off as R^{-3}. In the limit of no damping, the decay is governed by the natural linewidths of atom and oscillator, which have been ignored in this calculation. Thus, taking the limit $\gamma \to 0$ in the results for the atom in an excited state is not physically meaningful.

The techniques developed here can be transferred almost unaltered to the problem of an atom interacting with an assembly of weakly coupled oscillators. Furthermore, the present methods can be built upon for devising a strategy for dealing with strongly coupled oscillators which model solid dielectric bodies. In this case, one can dress the electromagnetic-field propagator and thus take into account the presence of the dielectric not perturbatively but exactly to all orders of the perturbation theory.

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APPENDIX: THE RETARDED PROPAGATOR OF THE DRESSED SYSTEM OSCILLATOR

1. Derivation of the propagator

The retarded propagator is defined as

$$K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t') = -\frac{i}{\hbar}\theta(t-t')\langle 0|[X_i(\mathbf{r},t),X_j(\mathbf{r}',t')]|0\rangle.$$

The dynamics of field $X_i(\mathbf{r},t)$ is described by the Hamiltonian H_{O-R}, Eq. (3). Hamilton's equations read

$$\dot{X}_i(\mathbf{r},t) = \frac{\delta H_{O-R}}{\delta P_i(\mathbf{r},t)} \equiv \frac{P_i(\mathbf{r},t)}{\mathcal{M}}, \tag{A1}$$

$$\dot{P}_i(\mathbf{r},t) = -\frac{\delta H_{O-R}}{\delta X_i(\mathbf{r},t)} \equiv -\mathcal{M}\omega_0^2 X_i(\mathbf{r},t) + \int_0^\infty d\nu \rho_\nu \nu^2 (Y_i(r,t,\nu) - X_i(\mathbf{r},t)). \tag{A2}$$

Differentiating $K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';t,t')$ with respect to time t and using Eq. (A1) gives

$$\frac{\partial}{\partial t}K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t') = -\frac{i}{\hbar}\theta(t-t')\left\langle 0\left|\left[\frac{P_i(\mathbf{r},t)}{\mathcal{M}},X_j(\mathbf{r}',t')\right]\right|0\right\rangle - \frac{i}{\hbar}\delta(t-t')\langle 0|[X_i(\mathbf{r},t),X_j(\mathbf{r}',t')]|0\rangle.$$

The second term vanishes because the equal-time commutator of the field with itself is zero. Differentiating with respect to t once more leads to

$$\frac{\partial^2}{\partial t^2}K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t') = -\frac{i}{\hbar\mathcal{M}}\theta(t-t') \times\left\langle 0\left|\left[\frac{\partial P_i(\mathbf{r},t)}{\partial t},X_j(\mathbf{r}',t')\right]\right|0\right\rangle - \frac{i}{\hbar}\delta(t-t') \times\left\langle 0\left|\left[\frac{P_i(\mathbf{r},t)}{\mathcal{M}},X_j(\mathbf{r}',t')\right]\right|0\right\rangle.$$

Now we use Eq. (A2) for rewriting the first term and the canonical commutation relations for simplifying the second term and obtain

$$\frac{\partial^2}{\partial t^2}K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t') = \frac{i\omega_1^2}{\hbar}\theta(t-t')\langle 0|[X_i(\mathbf{r},t),X_j(\mathbf{r}',t')]|0\rangle - \frac{i}{\hbar\mathcal{M}}\theta(t-t')\int_0^\infty d\nu\rho_\nu\nu^2 \times\langle 0|[Y_i(\mathbf{r},t,\nu),X_j(\mathbf{r}',t')]|0\rangle - \frac{1}{\mathcal{M}}\delta(t-t')\delta_{ij}\delta(\mathbf{r}-\mathbf{r}').$$

The shifted frequency ω_1 has been defined in Eq. (14) in the main text. We can rearrange the last equation to read

$$\frac{\partial^2}{\partial t^2}K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t') + \omega_1^2 K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t') = \frac{1}{\mathcal{M}}\int_0^\infty d\nu\rho_\nu\nu^2\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu) - \frac{1}{\mathcal{M}}\delta(t-t')\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'), \tag{A3}$$

where we have defined

$$\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu) = -\frac{i}{\hbar}\theta(t-t')\langle 0|[Y_i(\mathbf{r},t,\nu),X_j(\mathbf{r}',t')]|0\rangle.$$

Proceeding along exactly the same lines as for K^{ret} we can derive an equation for $\mathcal{F}$. Using the Hamilton equations of motion for reservoir fields,

$$\dot{Y}_i(\mathbf{r},t,\nu) = \frac{\delta H_{O-R}}{\delta Z_i(\mathbf{r},t,\nu)} \equiv \frac{Z_i(\mathbf{r},t,\nu)}{\rho_\nu},$$

$$\dot{Z}_i(\mathbf{r},t,\nu) = -\frac{\delta H_{O-R}}{\delta Y_i(\mathbf{r},t,\nu)} \equiv -\rho_\nu\nu^2[Y_i(\mathbf{r},t,\nu)-X_i(\mathbf{r},t)],$$

we derive

$$\frac{\partial^2}{\partial t^2}\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu) + \nu^2\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu) = \nu^2 K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t').$$

We now Fourier transform with respect to $t-t'$ and obtain

$$\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',\nu;\omega) = \frac{\nu^2}{\nu^2-\omega^2-i\varepsilon\omega}K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';\omega).$$

The infinitesimal imaginary term in the denominator is there to shift the poles of $\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',\nu;\omega)$ slightly into the lower half of the complex ω plane. This is necessary since the θ function in $\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',t,t',\nu)$ demands it to be zero for $t-t'<0$ and thus $\mathcal{F}_{ij}(\mathbf{r},\mathbf{r}',\nu;\omega)$ to be analytic in the upper half plane. We substitute the result into the Fourier transform of Eq. (A3),

$$(\omega_1^2-\omega^2)K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';\omega) = \frac{1}{\mathcal{M}}\int_0^\infty d\nu\rho_\nu\frac{\nu^4}{\nu^2-\omega^2-i\varepsilon\omega}K_{ij}^{\mathrm{ret}} \times(\mathbf{r},\mathbf{r}';\omega) - \frac{1}{\mathcal{M}}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}').$$

Resubstituting ω_1^2 from Eq. (14) we finally arrive at

$$K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';\omega) = \delta_{ij}\delta(\mathbf{r}-\mathbf{r}')K^{\mathrm{ret}}(\omega) \quad \text{with} \quad K^{\mathrm{ret}}(\omega) = \left[\mathcal{M}(\omega^2-\omega_0^2)-\omega^2\int_0^\infty d\nu\frac{\rho_\nu\nu^2}{\omega^2-\nu^2+i\varepsilon\omega}\right]^{-1}. \tag{A4}$$

2. Analytic properties of the propagator in ω space

For $K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}',t,t')$ to be retarded, i.e., to vanish for $t<t'$ its Fourier transform $K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';\omega)$ must not have poles in the upper half of the complex ω plane. We show that this is indeed the case, provided the function ρ_ν for real ν is real, positive, even, and has no poles on the real axis. The first two of these requirements are important also for other reasons: ρ_ν must be real to ensure the Hermiticity of the Hamiltonian, and it must be positive so that the Hamiltonian has a spectrum that is bounded from below, i.e., to guarantee the existence of a ground state.

We start by considering the analytic properties of the ν integral in the denominator of Eq. (A4). For even ρ_ν, we can extend the ν integration along the whole of the real axis and split the denominator into partial fractions

$$\omega^2\int_0^\infty d\nu\frac{\rho_\nu\nu^2}{\omega^2-\nu^2+i\varepsilon\omega} = \frac{\omega}{4}\int_{-\infty}^\infty d\nu\rho_\nu\nu^2 \times\left(\frac{1}{\omega+\nu+i\varepsilon}+\frac{1}{\omega-\nu+i\varepsilon}\right) = \frac{\omega}{2}\int_{-\infty}^\infty d\nu\frac{\rho_\nu\nu^2}{\omega-\nu+i\varepsilon}.$$

Thus, this integral as a function of ω has a cut just below the real axis but is analytic in the upper half plane.

To study the analytic properties of the whole of $K^{\mathrm{ret}}(\omega)$ in the upper half plane, we split the frequency into real and imarginary parts, $\omega=\omega'+i\omega''$ with $\omega''>0$. If the real part

ω' is nonzero, $K^{\mathrm{ret}}(\omega'+i\omega'')$ cannot have a pole, as one can see by examining the imaginary part of its inverse,

$$\mathrm{Im}\left(\frac{1}{K^{\mathrm{ret}}(\omega'+i\omega'')}\right)=2\mathcal{M}\omega'\omega''+\frac{\omega''}{2}\int_{-\infty}^{\infty}d\nu\,\frac{\rho_\nu\nu^3}{(\nu-\omega')^2+\omega''^2}.$$

The denominator of the ν integral gives rise to a Lorentzian peak around $\nu=\omega'$. Multiplied by ν^3 this peak gets weighed positively for positive ω' and negatively for negative ω'. Therefore, if ρ_ν is an even and positive function the integral must be positive for positive ω' and negative for negative ω'. The same is true trivially for the first term and, consequently, the inverse of $K^{\mathrm{ret}}(\omega)$ cannot be zero if ω has a nonzero real part. If $K^{\mathrm{ret}}(\omega)$ has any poles at all in the upper half plane, then they must lie on the imaginary axis. However, on the imaginary axis we find

$$\frac{1}{K^{\mathrm{ret}}(i\omega'')}=-\mathcal{M}(\omega''^2+\omega_0^2)+\frac{i\omega''}{2}\int_{-\infty}^{\infty}d\nu\,\frac{\rho_\nu\nu^2}{\nu^2+\omega''^2}(\nu+i\omega'').$$

For even ρ_ν the imaginary part of this expression is zero, but its real part is negative. Thus, $K_{ij}^{\mathrm{ret}}(\mathbf{r},\mathbf{r}';\omega)$ cannot have any poles in the upper half of the complex ω plane.

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